Asymptotically (anti)-de Sitter solutions in Gauss-Bonnet gravity without a cosmological constant

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In this paper we show that one can have asymptotically de Sitter (dS), anti-de Sitter (AdS) and flat solutions in Gauss-Bonnet gravity without any need to a cosmological constant term in field equations. First, we introduce static solutions whose 3-surfaces at fixed r and t have constant positive ($k = 1$), negative ($k = -1$), or zero ($k = 0$) curvature. We show that for $k = \pm 1$, one can have asymptotically dS, AdS and flat spacetimes, while for the case of $k = 0$, one has only asymptotically AdS solutions. Some of these solutions present naked singularities, while some others are black hole or topological black hole solutions. We also find that the geometrical mass of these 5-dimensional spacetimes is $m + 2\alpha\,|k|$, which is different from the geometrical mass, m, of the solutions of Einstein gravity. This feature occurs only for the 5-dimensional solutions, and is not repeated for the solutions of Gauss-Bonnet gravity in higher dimensions. We also add angular momentum to the static solutions with $k = 0$, and introduce the asymptotically AdS charged rotating solutions of Gauss-Bonnet gravity. Finally, we introduce a class of solutions which yields an asymptotically AdS spacetime with a longitudinal magnetic field which presents a naked singularity, and generalize it to the case of magnetic rotating solutions with two rotation parameters.

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I. INTRODUCTION

It seems established that at the present epoch the Universe expands with acceleration. This follows directly from the observation of high red-shift supernova [1] and indirectly from the measurement of angular fluctuations of cosmic microwave background fluctuations [2]. These astrophysical data have created a great deal of attention to the asymptotically de Sitter (dS) spacetimes. On the other hand asymptotically anti-de Sitter (AdS) spacetimes continue to attract more attention due to the fact that there is a correspondence between supergravity (the low-energy limit of string theory) in $(n+1)$-dimensional asymptotically AdS spacetimes and conformal field theory (CFT) living on an n-dimensional boundary known as the AdS/CFT correspondence

The simplest way of having an asymptotically (A)dS spacetime is to add a cosmological constant term to the right hand side of Einstein equation. However, the cosmological constant meets its well known cosmological, fine tuning and coincidence problems [3]. Thus, it seems natural to get rid of the cosmological constant and look for an alternative candidate for it. In the context of classical theory of gravity, the second way of having an asymptotically (A)dS spacetime is to add higher curvature terms to the left hand side of Einstein equation. The way that I deal with the asymptotically (A)dS spacetime is the latter one. Indeed, it seems natural to reconsider the left hand side of Einstein equation, if one intends to investigate classical gravity in higher dimensions.

The possibility that spacetime may have more than four dimensions is now a standard assumption in high energy physics. From a cosmological point of view, our observable Universe may be viewed as a brane embedded into a higher dimensional spacetime. The idea of brane cosmology is also consistent with string theory, which suggests that matter and gauge interaction (described by an open string) may be localized on a brane, embedded into a higher dimensional spacetime. The field represented by closed strings, in particular, gravity, propagate in the whole of spacetime.

This underscores the need to consider gravity in higher dimensions. In this context one may use another consistent theory of gravity in any dimension with a more general action. This action may be written, for example, through the use of string theory. The effect of string theory on classical gravitational physics is usually investigated by means of a low energy effective action which describes gravity at the classical level [5]. This effective action consists

of the Einstein-Hilbert action plus curvature-squared terms and higher powers as well, and in general give rise to fourth order field equations and bring in ghosts. However, if the effective action contains the higher powers of curvature in particular combinations, then only second order field equations are produced and consequently no ghosts arise [6]. The effective action obtained by this argument is precisely of the form proposed by Lovelock [7]. The appearance of higher derivative gravitational terms can be seen also in the renormalization of quantum field theory in curved spacetime [8].

In this paper we want to restrict ourself to the first two terms of Lovelock gravity, which are the Einstein-Hilbert and the Gauss-Bonnet terms. The latter term appears naturally in the next-to-leading order term of the heterotic string effective action and plays a fundamental role in Chern-Simons gravitational theories [9]. From a geometric point of view, the combination of the Einstein-Gauss-Bonnet terms constitutes, for five-dimensional spacetimes, the most general Lagrangian producing second order field equations, as in the four-dimensional gravity where the Einstein-Hilbert action is the most general Lagrangian producing second order field equations [10]. However, Gauss-Bonnet term is topological in 4-dimensions, and hence has no dynamics. Indeed, if this term had made a nontrivial contribution in 4-dimensions, then it would have conflicted with the $1/r$ character of the potential because of the presence of $(\nabla\phi)^4$ terms in the equation [11].

Thus, the problems with the cosmological constant, the need to go to higher dimensional spacetime, and the interest in asymptotically (A)dS spacetimes provide a strong motivation for considering asymptotically (A)dS solutions of the Einstein-Gauss-Bonnet gravity without cosmological constant. Recently I introduced a model for Universe in Gauss-Bonnet gravity without a cosmological constant, which is asymptotically de Sitter. In that model, one does not need to assume any kind of exotic dark energy, in order to explain the acceleration of the expanding Universe [12]. Most of the solutions of Gauss-Bonnet gravity which have been found till now are the solutions with nonzero cosmological constant. Static spherically symmetric black hole solutions of the Gauss-Bonnet gravity were found in Ref. [13]. Black hole solutions with nontrivial topology were also studied in Refs. [14, 15, 16]. The thermodynamics of charged static spherically symmetric black hole solutions was considered in [17]. All of these known solutions are static. Recently I introduced two classes of asymptotically anti-de Sitter rotating solutions in the Einstein-Gauss-Bonnet gravity and considered their thermodynamics [18, 19]. Also, the linearized gravity on a single de Sitter

brane in Gauss-Bonnet theory has been investigated [20].

The outline of our paper is as follows. We give a brief review of the field equations in Sec. II. In Sec. III, we introduce the static solutions of Gauss-Bonnet gravity without a cosmological constant term in the presence of electromagnetic field, and show that these solutions generate asymptotically (anti)-de Sitter and flat spacetimes. In Sec. IV, we find two classes of asymptotically AdS rotating solutions . We finish our paper with some concluding remarks.

II. FIELD EQUATIONS IN GAUSS-BONNET GRAVITY WITHOUT A COSMOLOGICAL CONSTANT

The most fundamental assumption in standard general relativity is the requirement that the field equations be generally covariant and contain at most second order derivative of the metric. Based on this principle, the most general classical theory of gravitation in five dimensions is the Einstein-Gauss-Bonnet gravity. The gravitational action of this theory in five dimensions for the spacetime ($\mathcal{M}$, $g_{\mu\nu}$) can be written as

$$I_G = \frac{1}{2}\int_{\mathcal{M}} dx^5 \sqrt{-g}[R + \alpha(R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2) + F_{\mu\nu}F^{\mu\nu}], \tag{1}$$

where R, $R_{\mu\nu\rho\sigma}$, and $R_{\mu\nu}$ are the Ricci scalar and Riemann and Ricci tensors of the spacetime, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor field, A_μ is the vector potential, and α is the Gauss-Bonnet coefficient with dimension (length)^2. Of course, one may add a constant term to the above Lagrangian, playing the role of cosmological constant term. But, as we mentioned before, this creates its own problems and therefore we don't disturb ourself with it. Indeed, here we want to obtain asymptotically (A)dS solutions without a cosmological constant term. Varying the action over the metric tensor $g_{\mu\nu}$ and electromagnetic field $F_{\mu\nu}$, the equations of gravitational and electromagnetic fields are obtained as

$$\begin{aligned} &R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \alpha\{\frac{1}{2}g_{\mu\nu}(R_{\kappa\lambda\rho\sigma}R^{\kappa\lambda\rho\sigma} - 4R_{\rho\sigma}R^{\rho\sigma} + R^2) \\ &-2RR_{\mu\nu} + 4R_{\mu\lambda}R^{\lambda}{}_{\nu} + 4R^{\rho\sigma}R_{\mu\rho\nu\sigma} - 2R_{\mu}{}^{\rho\sigma\lambda}R_{\nu\rho\sigma\lambda}\} = T_{\mu\nu}, \end{aligned} \tag{2}$$

$$\nabla_\mu F_{\mu\nu} = 0, \tag{3}$$

where $T_{\mu\nu}$ is the electromagnetic stress tensor

$$T_{\mu\nu} = 2F^{\lambda}{}_{\mu}F_{\lambda\nu} - \frac{1}{2}F_{\lambda\sigma}F^{\lambda\sigma}g_{\mu\nu}. \tag{4}$$

Equation (2) does not contain the derivative of the curvatures, and therefore the derivatives of the metric higher than two do not appear. Thus, the Gauss-Bonnet gravity is a special case of higher derivative gravity.

III. THE STATIC SOLUTIONS

Here we want to obtain the 5-dimensional static solutions of Eq. (2)-(4), which are asymptotically (anti)-de sitter or flat. We assume that the metric has the following form:

$$ds^2 = -f(r')dt^2 + \frac{dr'^2}{f(r')} + r'^2 d\Omega^2, \tag{5}$$

where $d\Omega^2$ is the metric of a 3-dimensional hypersurface with constant curvature $6k$ given as

$$\begin{aligned} d\Omega^2 &= d\theta^2 + \sin^2\theta(d\phi^2 + \sin^2\phi d\psi^2); \qquad k = 1, &(6)\\ &= d\theta^2 + \sinh^2\theta(d\phi^2 + \sin^2\phi d\psi^2); \quad k = -1, &(7)\\ &= \alpha^{-1}dx^2 + \sum_{i=1}^{2} d\phi_i^2; \qquad k = 0. &(8)\end{aligned}$$

Note that the coordinates x has the dimension of length, while the angular coordinates θ, ϕ, ψ and ϕ_i's are dimensionless as usual. The coordinates θ and ϕ lies in the interval $[0, \pi]$ and ψ and ϕ_i's range $0 \leq \phi_i < 2\pi$. The assumption that there exist a charged q at $r = 0$ (q is a point charge for $k = \pm 1$, and is the charge density of a line charge for $k = 0$ cases respectively) means that the vector potential may be written as

$$A_\mu = h(r')\delta_\mu^0. \tag{9}$$

The functions $f(r')$ and $h(r')$ may be obtained by solving the field equations (2)-(4). Using Eq. (3) one obtains

$$r'\frac{\partial^2 h}{\partial r'^2} + 3\frac{\partial h}{\partial r'} = 0. \tag{10}$$

Thus, $h(r') = -C_1/r'^2$, where C_1 is an arbitrary real constant. If one uses the Gauss law for the electric field, then he obtains $2C_1 = q$. To find the function $f(r)$, one may use any

components of Eq. (2). The simplest equation is the $r'r'$ component of these equations which can be written as

$$[12\alpha r'^3(1-f)+3r'^5]\frac{df}{dr'}-6r'^4(1-f)+2q^2=0. \tag{11}$$

The solutions of Eq. (11) can be written as

$$f(r')=k+\frac{r'^2}{4\alpha}\pm\sqrt{\frac{r'^4}{16\alpha^2}+\left(|k|+\frac{m}{2\alpha}\right)-\frac{q^2}{6\alpha r'^2}}, \tag{12}$$

where m is an arbitrary constant. Also, it is remarkable to note that for large values of r, the function $f(r)$ can be written as:

$$f_\infty(r')=k+\frac{r'^2}{4\alpha}(1\pm 1)\pm\frac{m+2\,|k|\,\alpha}{r'^2}\mp\frac{q^2}{3r'^4}, \tag{13}$$

which shows that the geometrical mass of the spacetime is $m+2\alpha\,|k|$. Thus, the mass of a five-dimensional spacetime in Gauss Bonnet gravity for $k=\pm1$, differs from that of Einstein gravity by a term which is proportional to $6\alpha\,|k|$. Note that the Gauss-Bonnet term decreases the mass of the spacetime for negative α, and increases the mass for positive α. It is worthwhile to mention that this occurs only for the five-dimensional spacetime. For higher-dimensional solutions in Gauss-Bonnet gravity $(n+1>5)$ the function $f(r)$ is

$$f(r')=k+\frac{r'^2}{2(n-2)(n-3)\alpha}\left(1\pm\sqrt{1+\frac{4(n-2)(n-3)\alpha m}{r'^{n-4}}-\frac{4(n-2)(n-3)^2\alpha q^2}{(n-1)r'^{2n-6}}}\right). \tag{14}$$

Equation (14) for large values of r' becomes:

$$f_\infty(r')=k+\frac{r'^2}{4\alpha}(1\pm 1)\pm\frac{m}{r'^{(n-2)}}\mp\frac{(n-3)q^2}{(n-1)r'^{(2n-4)}}, \tag{15}$$

which shows that the geometrical mass of the spacetime is the same as that of Einstein gravity.

One should note that the function $f(r')$ in Eq. (12) is imaginary for $r'<r_0$ and real for $r'>r_0$, where r_0 is the largest real solution of

$$3r_0^6+24\alpha(m+2\,|k|\,\alpha)r_0^2-8\alpha q^2=0. \tag{16}$$

Of course, one may note that Eq. (16) has real solution provided $\alpha>0$, or $32\alpha(m+2\,|k|\,\alpha)^3+3q^4<0$ for negative α. Thus, one cannot extend the spacetime to the region $r'<r_0$. To get rid of this incorrect extension, we introduce the new radial coordinate r as

$$r^2=r'^2-r_0^2\Rightarrow dr'^2=\frac{r^2}{r^2+r_0^2}dr^2. \tag{17}$$

With this new coordinate, the metric (5) and (12) become

$$ds^2 \;=\; -f(r)dt^2 + \frac{r^2}{r^2+r_0^2}\frac{dr^2}{f(r)} + (r^2+r_0^2)d\Omega^2, \tag{18}$$

$$f(r) \;=\; k + \frac{r^2+r_0^2}{4\alpha} \pm \sqrt{\left(\frac{r^2+r_0^2}{4\alpha}\right)^2 + \left(|k| + \frac{m}{2\alpha}\right) - \frac{q^2}{6\alpha(r^2+r_0^2)}}, \tag{19}$$

and the vector potential is $A_\mu = -q/(2\sqrt{r^2+r_+^2})\delta_\mu^0$. Of course, one may ask for the completeness of the spacetime with $r \geq 0$. It is easy to see that the spacetime described by Eq. (18) is both null and timelike geodesically complete for $r \geq 0$ [19].

In order to study the general structure of these solutions, we first look for the curvature singularities. It is easy to show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r = 0$ and therefore there is an essential singularity located at $r = 0$. As one can see from Eq. (19), the solution has two branches with "$-$" and "+" signs. We discuss them in the following subsections.

A. Asymptotically de Sitter solutions

We first investigate the "+" sign branch of $f(r)$ in Eq. (19) with $k = 1$. In this case α should be nonzero, but can have negative or positive values. The negative GB coefficient has been considered recently [12, 21]. If $\alpha < 0$, then the metric of Eqs. (18) and (19) has two inner and outer horizons located at

$$r_\pm = \left\{\frac{1}{6}\left[3m \pm \sqrt{9m^2 - 12q^2}\right] - r_0^2\right\}^{1/2}, \tag{20}$$

provided $9m^2 > 12q^2$. Thus, one encounters with an asymptoticalle dS black hole.

One can show that the Ricci scalar of the spacetime is $10/|\alpha|$ as r goes to infinity, and therefore the spacetime is asymptotically de Sitter. Thus, in the Gauss-Bonnet gravity, one can have asymptotically de Sitter black hole without any need to a cosmological constant term in the field equations. It is remarkable to note that there exist no asymptotically de Sitter solutions for $k = 0$, and -1.

B. Asymptotically anti-de Sitter solutions

Now, we consider the metric of Eqs. (18) and (19) for the "+" sign branch of $f(r)$ with positive values of α . The Ricci scalar of the solution is $-10/\alpha$, and therefore the spacetime

is asymptotically anti-de Sitter. For the case of $k = 0$ and 1; $f(r) > 0$ for all values of $0 \leq r < \infty$, and therefore this metric presents a naked singularity. While for the case of $k = -1$, the function $f(r)$ in Eq. (19) has a zero at $r = \{1/6[\sqrt{9m^2 + 12q^2} - 3m] - r_0^2\}^{1/2}$, and therefore we have an asymptotically AdS topological black hole.

Note that there is no asymptotically anti-de Sitter nontopological black hole in Gauss-Bonnet gravity without a cosmological constant. This feature is different from the case of Einstein or Gauss-Bonnet gravity with cosmological constant, which one has an asymptotically anti-de Sitter black hole in the latter cases.

C. Asymptotically flat solutions

Now we discuss the branch of $f(r)$ in Eq. (19) with "$-$" sign. One may note that for $k = 0$, the function $f(r)$ goes to 0 as r goes to infinity and therefore it is not acceptable. For $k = 1$, one may note that $f(r) \to 1$ as r goes to infinity and therefore the metric (18) and (19) is asymptotically flat. In this case for $\alpha > 0$, the metric of Eqs. (18) and (19) has two inner and outer horizons located at r_- and r_+, provided $9m^2 > 12q^2$. In the case that $9m = 12q^2$, we will have an extreme black hole, and for the case of $9m^2 < 12q^2$, one encounters with a naked singularity. It is remarkable to note that for large values of r, the function $f(r)$ can be written as:

$$f(r) \simeq 1 - \frac{m + 2\alpha}{3r^2} + \frac{q^2}{3r^4}, \tag{21}$$

which shows that the spacetime behaves like a Reissner-Nordstrum black hole with mass parameter $(m + 2\alpha)/3$. Thus, one may conclude that the mass of asymptotically flat black holes in Gauss-Bonnet gravity is more than the mass of asymptotically flat black holes in Einstein gravity. For negative values of α, the metric presents a naked singularity, and in the limit of $\alpha \to 0$, the spacetime is exactly asymptotically flat Reissner-Nordstrum spacetime as one expected. For $k = -1$ and $\alpha < 0$, this branch of $f(r)$ presents an asymptotically flat spacetime with naked sinqularity and a cosmological horizon.

IV. ROTATING SOLUTIONS

Here we consider two classes of rotating solutions in Gauss-Bonnet gravity without a cosmological constant for which the hypersurface of constant r and t are flat. As we have

seen in the last section, one can only have asymptotically AdS static solutions of these types.

A. Charged rotating solutions

First, we endow our spacetime solution (18) and (8) with a global rotation. The rotation group in $(n+1)$-dimensions is $SO(n)$ and therefore the number of independent rotation parameters for a localized object is equal to the number of Casimir operators, which is $[n/2]$, where $[z]$ is the integer part of z. Therefore for the case of a five-dimensional spacetime, one can have at most two rotation parameters. It is easy to show that the metric (18) and (8) with two rotation parameters a_1 and a_2 can be written as [18]

$$\begin{aligned}
ds^2 &= -f(r)\left(\Xi dt - \sum_{i=1}^{2} a_i d\phi_i\right)^2 + \frac{r^2}{r^2+r_0^2}\frac{dr^2}{f(r)} \\
&\quad \frac{r^2+r_0^2}{\alpha}\left\{\alpha^{-1}\sum_{i=1}^{2}(a_i dt - \Xi\alpha d\phi_i)^2 - (a_1 d\phi_2 - a_2 d\phi_1)^2 + dx^2\right\}, \\
\Xi^2 &= 1 + \alpha^{-1}\sum_{i=1}^{2} a_i^2, \\
A_\mu &= \frac{q}{2r^2}((\Xi\delta_\mu^0 - a_i\delta_\mu^i), \qquad \text{(no sum on i)},
\end{aligned} \tag{22}$$

where the functions $f(r)$ is given by Eq. (19).

B. Magnetic rotating solutions

Here we want to obtain the 5-dimensional solutions of Eqs. (2)-(4) which produce a longitudinal magnetic field normal to the $(r-\phi_1)$-plane. We assume that the metric has the following form:

$$ds^2 = -\frac{\rho^2}{\alpha}dt^2 + \frac{dr^2}{f(r)} + \alpha f(r) d\phi_1^2 + r^2 d\phi_2^2 + \frac{r^2}{\alpha}dx^2. \tag{23}$$

Again, the coordinates x have the dimension of length, while the angular coordinate ϕ_i are dimensionless as usual and ranges in $0 \leq \phi_i < 2\pi$. The motivation for this metric gauge $[g_{tt} \propto -r^2$ and $(g_{rr})^{-1} \propto g_{\phi_1\phi_1}]$ instead of the usual Schwarzschild gauge $[(g_{rr})^{-1} \propto g_{tt}$ and $g_{\phi\phi} \propto r^2]$ comes from the fact that we are looking for a magnetic solution instead of an electric one. First, we consider only the static solution. Since, we want to have a magnetic

field, one may assume that $A_\mu = h(r)\delta_\mu^{\phi_1}$. Using the field equations (2)-(4), one obtains

$$
\begin{aligned}
f(r) &= \frac{r'^2}{4\alpha} \pm \sqrt{\frac{r^4}{16\alpha^2} + \frac{m}{6\alpha} + \frac{q^2}{6\alpha r^2}}, && (24) \\
A_\mu &= \frac{q}{2\sqrt{\alpha} r^2}\delta_\mu^{\phi_1}. && (25)
\end{aligned}
$$

The only nonvanishing component of electromagnetic field is $F_{r\phi_1} = \sqrt{\alpha}q/r^3$, which is a longitudinal magnetic field normal to the $(r - \phi_1)$-plane. In order to study the general structure of these solutions, we first look for curvature singularities. It is easy to show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r = 0$ and therefore there is an essential singularity located at $r = 0$. As one can see from Eq. (24), the solution has two branches with "$-$" and "$+$" signs. Since the " $-$" signs branch goes to zero as r goes to infinity, therefore it cannot be accepted. The "$+$" signs branch is always positive, and therefore this spacetime has no horizon. Thus, the metric (23)-(25) presents a naked singularity.

Now we consider the most general magnetic rotating solution which can have two rotation parameter in five dimensions. It is easy to show that the following metric satisfies the field equations (2)-(4):

$$
\begin{aligned}
ds^2 &= -\frac{r^2}{\alpha}\left(\Xi dt - \sum_{i=1}^{2} a_i d\phi_i\right)^2 + \frac{f(r)}{\Xi^2 - 1}\sum_{i=1}^{2}\left((\Xi^2 - 1)dt - \Xi\sum_{i=1}^{2} a_i d\phi_i\right)^2 \\
&\quad - \frac{r^2}{\alpha(\Xi^2 - 1)}(a_1 d\phi_2 - a_2 d\phi_1)^2 + \frac{dr^2}{f(r)} + \frac{r^2}{\alpha}dx^2, \\
\Xi^2 &= \alpha^{-1}\sum_{i=1}^{2} a_i^2, \\
A_\mu &= \frac{q}{2r^2}((\Xi\delta_\mu^0 - a_i\delta_\mu^i), \qquad \text{(no sum on i)}. && (26)
\end{aligned}
$$

V. CLOSING REMARKS

In this paper we investigated the classical theory of gravity without cosmological constant. Indeed, we added the Gauss-Bonnet term to the Einstein action and introduced a few solutions of the field equations in the presence of an electromagnetic field. We found that one can have asymptotically de Sitter, flat or anti-de sitter solutions in Gauss-Bonnet gravity without any need to a cosmological constant term in gravitational field equations. First, we introduced static solutions whose 3-surfaces at fixed r and t have constant positive

($k = 1$), negative ($k = -1$), or zero curvature ($k = 0$). We encountered with two different branches for $f(r)$ in Eq. (19). For "+" sign branch, we showed that when $k = 1$, then one could have asymptotically dS and AdS solutions. Indeed, for negative α, the solution presented an asymptotically dS black hole with event (E) and cosmological (C) horizons (H) provided $9m^2 > 12q^2$, while for positive values of α, the spacetime was asymptotically AdS with a naked singularity (NS). For the case of zero k, one could have only asymptotically AdS solutions, while for $k = -1$ and positive α, one had an asymptotically AdS topological black hole. For "−" sign branch, the solutions were asymptotically flat. See for more details the following table:

Branch sign of f(r)	**k**	α	**Asymptotic behavior**	**Singularity**
+	1	−	dS	BH with E&CH ($9m^2 > 12q^2$)
+	1	+	AdS	NS
+	0	+	AdS	NS
+	−1	+	AdS	BH with EH
−	1	+	flat	BH with 2 H's ($9m^2 > 12q^2$)
−	1	+	flat	BH with EH ($9m^2 = 12q^2$)
−	1	+	flat	NS ($9m^2 < 12q^2$)
−	1	−	flat	NS
−	−1	−	flat	NS with CH

We found that the geometrical mass of these 5-dimensional spacetimes is $m + 2\alpha\,|k|$, which is different from the geometrical mass, m, of the solutions of Einstein gravity. It seems resonable to say that the Gauss-Bonnet term with negative coefficient α, acts as a negative mass or antigravity effect. This feature occured only for the 5-dimensional solutions, and was not repeated for the solutions of Gauss-Bonnet gravity in higher dimensions.

We also added angular momentum to the static solutions with $k = 0$, and introduced the asymptotically AdS charged rotating solutions of Gauss-Bonnet gravity with two rotation parameters. Finally, we found a class of solutions which yields an asymptotically AdS spacetime with a longitudinal magnetic field [the only nonzero component of the electromagnetic field is $F_{r\phi}$] generated by a static magnetic brane. We found that these solutions have curvature singularity at $r = 0$ without any horizons. We also introduced the magnetic

rotating solutions with two rotation parameters. In these spacetimes, when all the rotation parameters are zero (static case), the electric field vanishes, and therefore the brane has no net electric charge. For the spinning brane, when one or more rotation parameters are nonzero, the brane has a net electric charge density which is proportional to the magnitude of the rotation parameter given by $\sqrt{\Xi^2 - 1}$.

As stated before, the Gauss-Bonnet gravity is the most general gravitational field equation in five dimension. In higher dimension one should use more terms for action in Lovelock theory. The consideration of asymptotically (A)dS solutions in Lovelock gravity with more curvature terms remains to be carried out in future.

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